**mgs** | **The Manchester Grammar School**
Founded 1515

# Entrance Examination 2017
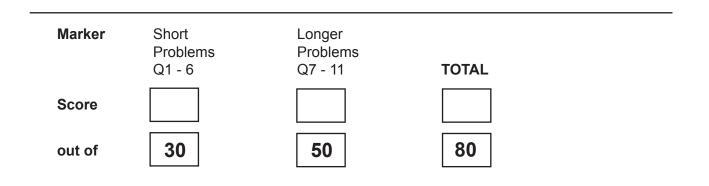# Arithmetic Section B

# 1 Hour

## Do not open this booklet until told to do so

## Calculators may not be used

**Write your names, school and candidate number in the spaces provided at the top of this page.**

For each question, show all your working in full, as this will be marked, and then write your answer clearly in the space provided. If you run out of space for an answer use the space provided at the end of this booklet, numbering your answers carefully.

You have 1 hour for this paper which is worth 80 marks.

| Marker | Short Problems Q1 - 6 | Longer Problems Q7 - 11 | TOTAL |
|---|---|---|---|
| Score | | | |
| out of | 30 | 50 | 80 |

1. When we multiply three consecutive numbers as shown below we obtain the following answer

$$17 \times 18 \times 19 = 5814$$

**Using this result,** work out the answers to these four questions

(a)  17 x 18 x 38 =

| 1a | |
|---|---|

(b)  9 x 19 x 34  =

| 1b | |
|---|---|

(c)  170 x 180 x 190 =

| 1c | |
|---|---|

(d)  5814 ÷ 51 ÷ 38 =

| 1d | |
|---|---|

**[4 marks]**

2. (a) The **average** height of three boys is 1.29 metres. A fourth boy Stephen, whose height is 1.21 metres, joins the group. Find the new **average** height of the group.

| 2a | | m |
|---|---|---|

(b) Two more boys, Richard and Nigel, join the group. The new **average** height of all six boys in the group is 1.28 metres. If Richard is 1.26 metres, how tall is Nigel?

| 2b | | m |
|---|---|---|

**[5 marks]**

3. The usual Olympic medal table, like the one used in Rio in 2016, simply ranks countries in order of the number of Gold medals won.

   However, the table below has been created by giving a point score to each type of medal won. The number of points awarded for each of the medals in this table is as follows:- 3 points for every Gold medal won, 2 points for each Silver medal won and 1 point for a Bronze medal won.

   **Complete the table below by filling in the missing <u>numbers of medals</u> using the point scores given to the Gold, Silver and Bronze medals. You need to know that the country of Esthopia won equal numbers of Gold, Silver and Bronze medals in order to complete the final line.**

| Country | Gold (3pts) | Silver (2pts) | Bronze (1pt) | Points (Pts) |
|---------|-------------|---------------|--------------|--------------|
| Abalasia | 17 | 10 | 8 | 79 |
| Brussland | 13 | 8 | 16 | |
| Ceeden | 14 | 9 | | 65 |
| Dorway | | 8 | 7 | 62 |
| Esthopia | | | | 60 |

**[5 marks]**

4. In a special code, words are replaced by the **product** of the **whole** numbers assigned to the letters. In the code, each letter is given a different number.

**For example:- if S = 3, P = 4 and Y = 6 then the word**

**SPY = 3 x 4 x 6 = 72**

**Using this same method of creating our special code,** work out the answers to the following **four** questions

(a) If TEE = 20, find the values of T and E, if **neither** of the letters has the value 1.

| 4a | T= |
|----|----|
|    | E= |

(b) Then with those values for T and E, if TEA = 70, find the value of A.

| 4b | A= |
|----|----|

(c) Now work out the value of the word SEAT with the letter values you have, including those in the example at the start.

| 4c | SEAT = |
|----|--------|

(d) Finally, if the value of the word FOAL = 504, work out the value of the word LOAF.

| 4d | LOAF = |
|----|--------|

**[5 marks]**

5. On a taxi journey with one particular taxi company, the fare is worked out using a set starting charge plus a charge for each quarter of a mile travelled (the QMC).

So if the starting charge is £1 and the QMC (charge for each quarter of a mile) is 50p then the total fare for a one mile journey is given by

**Total Fare = £1 + 4 x 50p = £3**

(a) If the Total Fare for a two and a half mile ($2\frac{1}{2}$ mile) journey at another taxi company is £9.60 and the QMC (charge for each quarter of a mile) is 80p, what is the starting charge?

| 5a | £ |
|----|---|

(b) If the starting charge at a third company is £2.20 and the Total Fare for a $6\frac{1}{4}$ mile journey is £12.20, what is the QMC?

| 5b | | p |
|----|---|---|

**[5 marks]**

6.  The firm *Owl Blocks* makes rectangular wooden blocks in many sizes. For all of their blocks the length of the block is always 2.5 times the width of the block. The height of the blocks isn't limited in any way.

    **For example, if the width of a block is 4cm then the length of the block would be 10cm**

    **because   4cm x 2.5  =  10cm**

    (a) Find the length of a block if its width is 7cm.

    | 6a | | cm |
    | --- | --- | --- |

    (b) Find the width of a different block if its length is 50cm.

    | 6b | | cm |
    | --- | --- | --- |

    (c) Find the length of a third block if the **perimeter** of one of its faces measured using its length and width is 42cm.

    | 6c | | cm |
    | --- | --- | --- |

**[6 marks]**

7. The distance, **d**, in metres, travelled by any vehicle accelerating at a rate **A**, in **t** seconds is given by the formula

$$d = A \times t^2 \div 2 \quad or \quad d = A \times t \times t \div 2$$

(a) Find the distance travelled by a car accelerating at a rate of 8 for 3 seconds.

| 7a | |
|---|---|
| | m |

(b) Find the distance travelled by a space rocket accelerating at a rate of 20 for **2 minutes.**

| 7b | |
|---|---|
| | m |

(c) A motorbike travels 270 metres in 6 seconds. What is its rate of acceleration?

| 7c | |
|---|---|

(d) A truck travels 50 metres while accelerating at a rate of 4. How long does this take?

| 7d | |
|---|---|
| | secs |

**[8 marks]**

8. At **"Fryers and Co"** a portion of chips costs £1.50, a fish costs £3.50 and a pie costs £2.00. Last Saturday evening, they sold 80 lots of fish and chips, 70 lots of pie and chips and 50 portions of chips on their own.

   (a)   Work out their total income that evening.

   | 8a | £ |
   |----|---|
   |    |   |

   The owner bought **two** 25 kilogram bags of potatoes which cost £40 each for the chips. He paid £2.00 for each of the fish he sold and £1.20 for each of the pies.

   (b)   How much did the food that the owner
         sold that evening cost him?

   | 8b | £ |
   |----|---|
   |    |   |

   The owner employs two staff for the evening, paying them £8 per hour each and they work from 4pm to 11pm. Other materials cost £10 for the whole evening.

   (c)   What are the **TOTAL** costs for the evening,
         including the food, the staff and the other materials?

   | 8c | £ |
   |----|---|
   |    |   |

   (d)   How much profit (Income  –  Total Costs) did the owner
         make that Saturday evening?

   | 8d | £ |
   |----|---|
   |    |   |

**[8 marks]**

9. The following table shows the first three rows of a table which has five columns. The **values** of all the entries on a particular row are the same but they are expressed in different forms.

| Row 1 | $1^3$ | 1 | 1 | $1^2$ | $1^2$ |
|---|---|---|---|---|---|
| Row 2 | $1^3 + 2^3$ | 1 + 8 | 9 | $3^2$ | $(1 + 2)^2$ |
| Row 3 | $1^3 + 2^3 + 3^3$ | 1 + 8 + 27 | 36 | $6^2$ | $(1 + 2 + 3)^2$ |
| Row 4 | a) | | | b) | |
| Row 5 | | c) | | | d) |
| Row 6 | | | e) | | |

**Without necessarily completing** the entire table, write down the **entry** that would be written in the space in the table denoted by each of the letters

(a)

9a [ ]

(b)

9b [ ]

(c)

9c [ ]

(d)

9d [ ]

(e)

9e [ ]

**[10 marks]**

10. In a game, John has 10 cards numbered 1 to 10 and five of his friends each pick two of the cards and look at the numbers on the cards they have chosen.

**The numbers on Andy's two cards add up to 10**

**The numbers on one of Bilal's cards is a factor of the number on his other card**

**The numbers on one of Cheryl's cards is the square of the number on the other card**

**The numbers on David's two cards are both prime**

**The difference between the numbers on Eleanor's two cards is 9**

Using the information in bold type above complete this table to show which two cards each person picked. Remember, each numbered card can only be placed in one box.

| John's friends | One card | Other Card |
|---|---|---|
| Andy | | |
| Bilal | | |
| Cheryl | | |
| David | | |
| Eleanor | | |

**[10 marks]**

11. The "Electric Light Organization" makes many electrical components including **Zisters** which control the power to any electrical appliance.
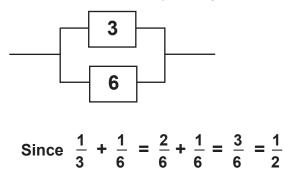
**This Zister** ─ | **3** | ─ **has a value of 3.**

Zisters can be put together in two different ways as follows

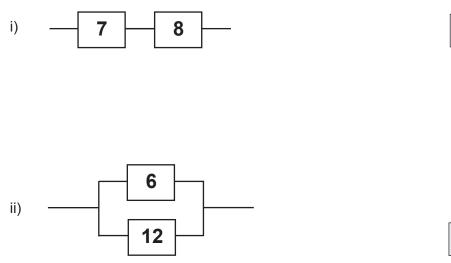In an **AFFTA**, the two values of the **Zisters** are added, so for example

─| **3** |─| **6** |─ **has a value of 9, because 3 + 6 = 9.**

In a **NEXTA**, the value has to be worked out by adding fractions as follows

| **3** |
| **6** |

**Since** $\dfrac{1}{3} + \dfrac{1}{6} = \dfrac{2}{6} + \dfrac{1}{6} = \dfrac{3}{6} = \dfrac{1}{2}$

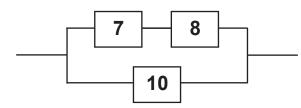**the value of this NEXTA is 2, taken from the <u>answer</u> to the fraction sum.**

(a) Work out the values of the following combinations of **Zisters** in these two questions using the methods in the examples above.
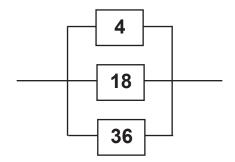
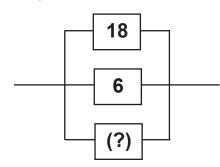i) ─| **7** |─| **8** |─

| 11ai | |

ii) ─| **6** | **12** |─

| 11aii | |

(b)  Work out the value of this combination of **AFFTA** and **NEXTA.**

```
    ┌──[ 7 ]──[ 8 ]──┐
────┤                 ├────
    └──────[ 10 ]─────┘
```

| 11b | |
|---|---|

(c)  Work out the value of this **triple NEXTA** by adding three fractions.

```
    ┌──[ 4 ]──┐
    ├──[ 18 ]─┤
────┤         ├────
    └──[ 36 ]─┘
```

| 11c | |
|---|---|

(d)  If this <u>**new**</u> **triple NEXTA** has a value of 3, work out the value of the missing **Zister** shown with (?)

```
    ┌──[ 18 ]──┐
    ├──[ 6 ]───┤
────┤          ├────
    └──[ (?) ]─┘
```
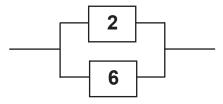
| 11d | |
|---|---|

(e)  Finally, here is a <u>**special**</u> **NEXTA.** Showing all your working, calculate its value.
(it is special because its value is not a whole number!)

```
    ┌──[ 2 ]──┐
────┤          ├────
    └──[ 6 ]──┘
```

| 11e | |
|---|---|

**[14 marks]**

# This is the end of the Examination

**Use any remaining time to check your work
or try any questions you have not answered.**