WESTMINSTER SCHOOL
THE CHALLENGE 2018

MATHEMATICS III

Wednesday 2nd May 2018

Time allowed: 1 hour 30 minutes

You may not use a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

1 Ravi scored 65 marks out of a total of 90 in French.
In Mathematics, he scored 80 marks out of a total of 110.
Did he get a higher percentage in French or Mathematics? Show clearly how you decide.

2 Initially, the amounts of money Alex, Ben and Chris have are in the ratio 7: 8 : 3. Then, Alex and Ben each give Chris £1; after that Ben and Chris have the same amount of money. How much should Ben and Chris each now give to Alex so that all three boys have the same amount of money?

3 A driver travels a total distance of 312 miles.

- She travels 90 miles on an A-road travelling an average of 36 miles per gallon of petrol.
- Then she takes the motorway, using 3·5 gallons of petrol while travelling an average of 60 miles per gallon of petrol.
- Then she drives through a town.

Over the whole journey, the driver travels an average of 48 miles per gallon of petrol.

In the town, how many miles was she travelling on average per gallon of petrol?

4 In a school, $\frac{5}{12}$ of the pupils are boys and $\frac{7}{12}$ are girls.

a Given that $\frac{2}{15}$ of the school are year 8 boys, what fraction of the boys are in year 8?

b Given also that $\frac{18}{35}$ of the girls are in year 8, what fraction of year 8 are boys?

5 In 2015, a shop made 25% more profit than in 2014, and in 2016, the shop made 25% more profit than in 2015. In 2017, the shop made the same profit as in 2014. What was the percentage reduction in the shop's profits between 2016 and 2017?

6 When Tom is training, he runs x miles and then cycles y miles.

One morning, he runs at $7\frac{1}{2}$ miles per hour, and cycles at $22\frac{1}{2}$ miles per hour. Altogether, his training takes one hour and twenty minutes.

One evening, he runs at 5 miles per hour and cycles at 18 miles per hour. Altogether his training takes one hour and fifty minutes.

Use an algebraic method to work out the values of x and y.

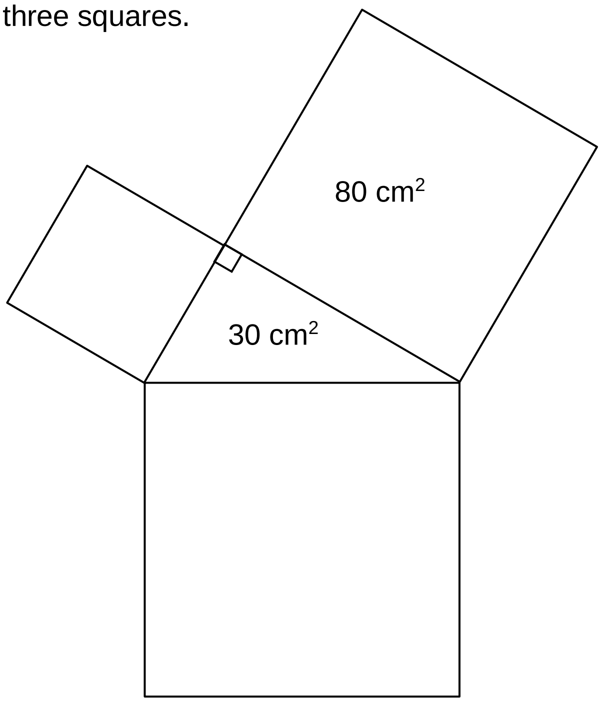
7 Two regular polygons are drawn: polygon A and polygon B.

- Polygon B has five times as many sides as polygon A
- The internal angle of polygon B is 9° larger than the internal angle of polygon A.

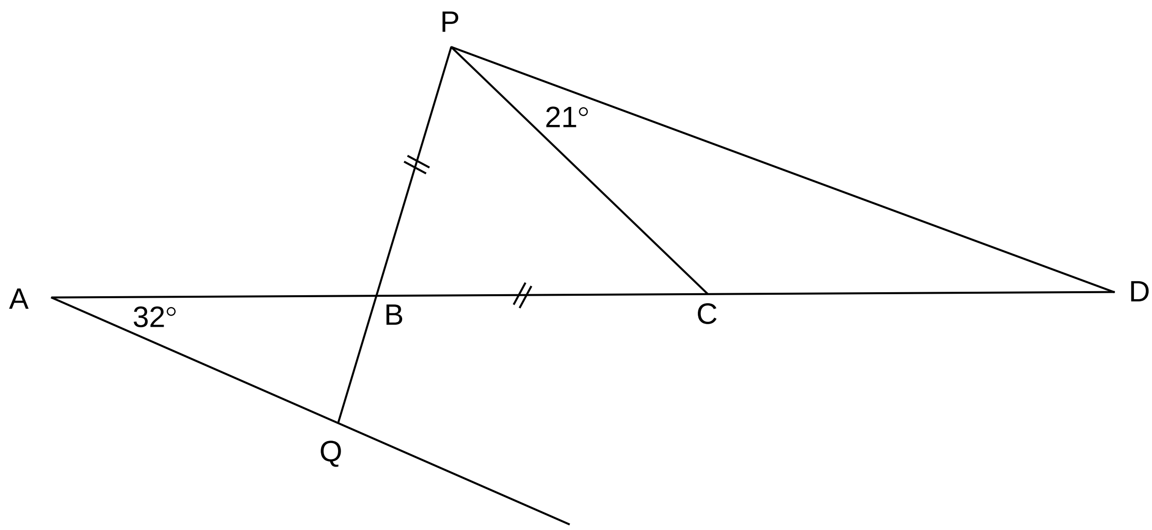
How many sides does each polygon have and what are their internal angles?

8 The diagram shows a right angled triangle and three squares.
One square has area 80 cm^2, as shown.
The triangle has area 30 cm^2.

a What is the area of the smaller square?
b What is the area of the largest square?



9 In the diagram, ABCD and PBQ are straight lines and triangle BPC is isosceles with BP = BC.



a Let angle CDP = x°. Find the following angles in terms of x, giving reasons for each step in your argument.
i Angle PBC.
ii Angle AQB.

b Prove that, if PD is parallel to AQ, then triangle ABQ is isosceles, with AB = AQ.

10 I have some tins of red paint and some tins of white paint, all of the same size, and I want to make some pink paint by mixing red and white paint. If I mix tins of red and white paint in the ratio 3 : 4, I can use all my tins of white paint and have one tin of red paint left over. If I mix tins of red and white paint in the ratio 4 : 5, I can use all my tins of red paint and have one tin of white paint left over.

In what ratio should I mix tins of red and white paint to have no tins of either colour left over?

11 **a** **i** Explain why $\sqrt{3^6 \times 7^2}$ is $3^3 \times 7$.

ii Simplify $\sqrt{5^{16}}$

iii Simplify $\sqrt[3]{3^3 \times 5^6}$

A square, or cube, number is defined as one that can be written as the square, or cube, of a *whole* number.

b For each of these numbers, written in index notation, state whether it is a square number but not a cube number; a cube number, but not a square number; both a square number and a cube number, or neither. Justify your answers.

i	$2^6 \times 3^2$	**ii**	$2^6 \times 5^{12}$	**iii**	$2^9 \times 10^3$
iv	$2^{12} \times 9^3$	**v**	$8^4 \times 7^3$	**vi**	$12^3 \times 3^5$

c If $p = 2^5 \times 3$, what is the smallest positive whole number you could multiply by p, so that the answer is a cube number?

d Given that $2n$ is a square number and $5n$ is a cube number what is the smallest positive whole number that n could be? Justify your answer.

e Given that $2m$ is a square number and $4m$ is a cube number, 2 is the smallest positive integer that m could be. What is the next smallest?

12 **a** Find all the positive whole numbers a and b which satisfy the following equation

$$\frac{a}{20} + \frac{b}{15} = \frac{11}{12}$$

and for which the fractions $\frac{a}{20}$ and $\frac{b}{15}$ are in their lowest terms.

b Find all the positive whole numbers c and d which satisfy the following equation

$$\frac{c}{21} + \frac{1}{d} = \frac{5}{14}$$

and for which the fraction $\frac{c}{21}$ is in its lowest terms.

HINT: First multiply both sides of the equation by 42.

TURN OVER FOR QUESTIONS 13 and 14

13 The symbol **M** means "multiply by 2" and the symbol **A** means "add 2".
These symbols can be combined in strings, so that **MMA** means "multiply by two, then multiply again by 2, then add 2". **MMA** applied to 7 gives $7 \times 2 \times 2 + 2 = 30$, and **MMA** applied to x gives $4x + 2$.

In part **a**, use only the symbols **M** and **A**.

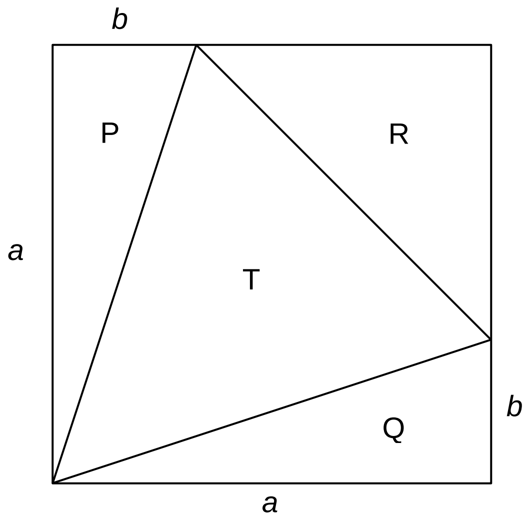
a **i** What is the result, in its simplest form, of applying the string **MMAAA** to x?
ii Find a string of *three* symbols which always has the same effect as **MAMAA**.
iii Find the shortest possible string which always gives the result $4x + 22$ when applied to x.

The symbol **D** means "divide by 2" and the symbol **S** means "subtract 2".
These symbols can also be combined in strings.

In part **b** you may use any of the symbols **M**, **A**, **D** or **S**.

b **i** What is the result, in its simplest form, of applying the string **SDAM** to x?
ii Find a string of *three* symbols which always has the same effect as **MSDA**.
iii Find the shortest possible string which always gives the result $4x + 3$ when applied to x.

14 The diagram below shows an isosceles triangle, T, in a square. The letters *a* and *b* represent some of the lengths (in centimetres) in the figure.



a Find, in terms of *a* and *b*, the areas of the three right angled triangles labelled P, Q and R.
b Hence find an expression for the area of the isosceles triangle T. Simplify your expression as far as possible.
c Find all the possible whole number values of *a* and *b* which make the area of T equal to 60 cm^2.