

# Eton College King's Scholarship Examination 2018

## MATHEMATICS B

(One and a half hours)

*Candidate Number:.....*

***Please write your candidate number on EVERY sheet.***

*Please answer on the paper in the spaces provided.*

*Each question is worth 10 marks.*

*Show all your working.*

*Answers without sufficient working may receive little or no credit.*

***The use of calculators is permitted.***

**Do not turn over until told to do so.**

1. a) Solve for  $x$ :

$$\frac{1}{3 + \frac{1}{x}} = \frac{3}{16}$$

b) Solve for  $x$ :

$$\frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} = \frac{5}{7}$$

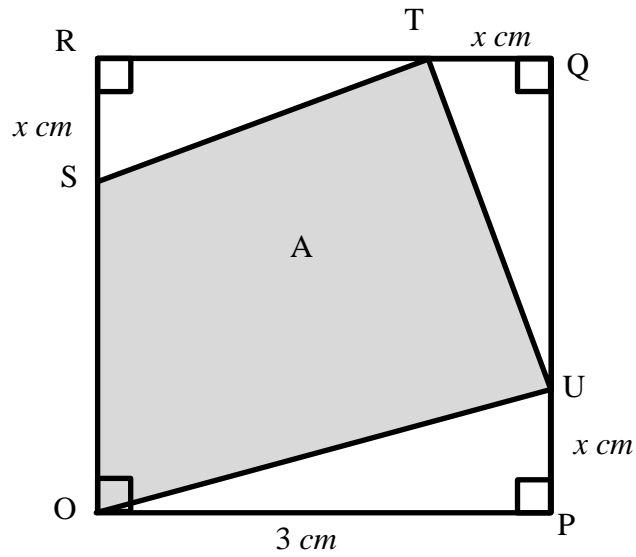
Notice that

$$\frac{5}{11} = \frac{1}{11/5} = \frac{1}{2 + \frac{1}{5}}$$

c) Find positive integers  $a$ ,  $b$ , and  $c$  such that

$$\frac{1}{a + \frac{1}{b + \frac{1}{c}}} = \frac{7}{17}$$

2.



$OPQR$  is a square of side length  $3\text{ cm}$ .  $PU = QT = RS = x\text{ cm}$ , where  $0 < x < 3$ . The shaded region  $OSTU$  has area  $A$ .

a) Show that  $A = x^2 - \frac{9}{2}x + 9$

b) Expand  $\left(x - \frac{9}{4}\right)^2$ , simplifying fully.

c) Hence find a number  $d$  such that  $A = \left(x - \frac{9}{4}\right)^2 + d$

d) For what value of  $x$  is  $A$  smallest, and what is this area?



- c) What is the second time that Jeff and Will pass each other?
- d) How far from line A are Jeff and Will when they pass each other for the third time?

4. a) Show that  $(1 + x + x^2)(1 - x) = 1 - x^3$

b) *Without* using a calculator and showing full working, use part a) to show that:

$$1 + \frac{2}{7} + \frac{4}{49} = \frac{1 - \frac{8}{343}}{1 - \frac{2}{7}}$$

c) Multiply out  $(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8)(1 - x)$  and simplify fully:

d) *Without* using a calculator and showing full working, evaluate the following sum, leaving your answer as a fraction:

$$\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \frac{64}{729} + \frac{128}{2187} + \frac{256}{6561}$$

5. Suppose  $x = 0.1\dot{2}\dot{3} = 0.1232323 \dots$

a) State the value of  $10x$

b) State the value of  $1000x$

c) Hence show that  $990x = 122$  and write  $x$  as a fraction in its lowest terms.

Suppose now that  $y = 17.\dot{a}\dot{b} = 17.ababab \dots$  where  $a$  and  $b$  are integers between 0 and 9.

d) By considering  $100y$ , show that  $y = \frac{1683+10a+b}{99}$

e) If  $z = 71.7\dot{c}\dot{d}$ , find integers  $m$  and  $n$  such that  $z = \frac{m+10c+d}{n}$



6.a) Express  $\frac{1}{x} + \frac{1}{y}$  as a single fraction.

b) Show  $x^2 + y^2 = (x + y)^2 - 2xy$

If  $xy = 5$  and  $x + y = 8$ :

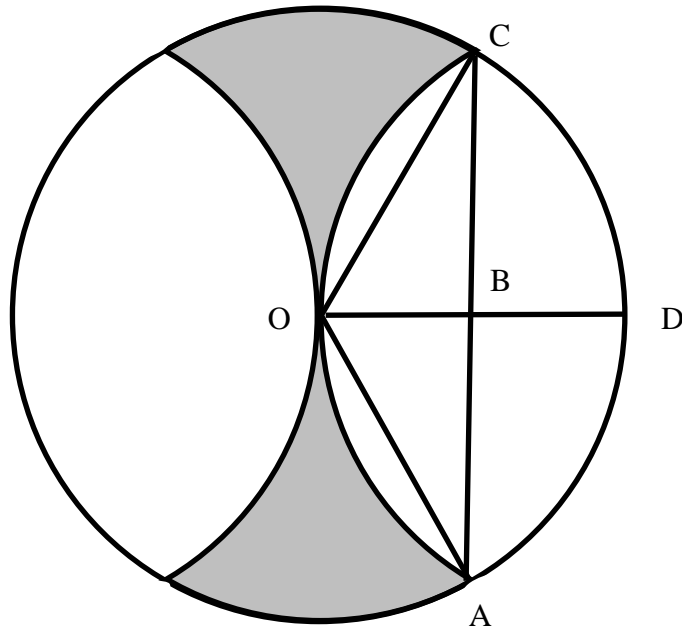
c) Calculate the value of  $\frac{1}{x^2} + \frac{1}{y^2}$ , leaving your answer as a fraction.

d) By considering  $(x + y)^3$ , calculate the value of  $\frac{1}{x^3} + \frac{1}{y^3}$ , leaving your answer as a fraction.

7. Throughout this question, leave all answers as exact expressions in terms of  $\pi$  and square roots, for example  $2(\pi + \sqrt{3})$  or  $\frac{2\pi}{\sqrt{5}}$  etc.

In the diagram below, the circle centred at O has radius 3cm. The curved line AOC is part of a circle, also of radius 3cm, centred at D. OD is perpendicular to AC.

**DIAGRAM 1**

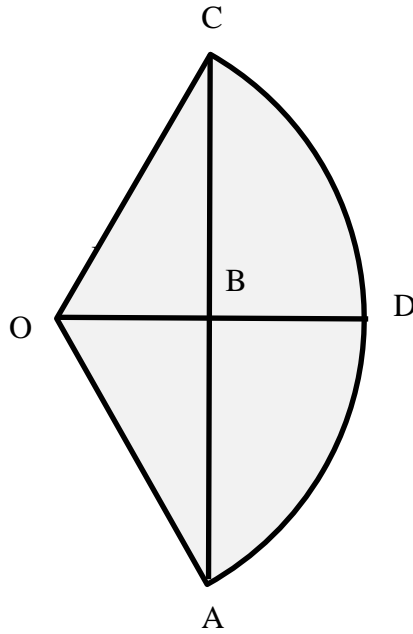


a) Calculate the length AB.

b) Calculate the area of triangle AOC.

c) Calculate the area of sector OADC in diagram 1 (indicated for reference in diagram 2 below).

**DIAGRAM 2**



d) Calculate the shaded area indicated in **diagram 1**.

8. Sarah has an accurate analogue wristwatch with a standard 12-hour face including hour and minute hands.

What is the smaller angle between the minute and hour hands when the time is:

a) 19:45?

b) 15:27?

c) What is the first time after 3pm that the angle between the two hands is  $119^\circ$ ?

d) Sarah starts her homework shortly after 5pm, when the angle between the hands is  $128^\circ$ . Her homework takes her more than 1 hour. When she finishes the homework, the angle between the hands is  $12^\circ$ . What is the smallest possible time it took Sarah to complete her homework?

END OF PAPER