MATHEMATICS B

(One and a half hours)

Candidate Number:..

Please answer on the paper in the spaces provided.

<u>Please write your candidate number on every sheet.</u>

Each question is worth 10 marks.

Show all your working.

The use of calculators is permittted.

<u>*ADDITIONAL MATERIALS:*</u>   *None*

Do not turn over until told to do so.

1.   a)   Show that $(k+1)^2 - k^2 = 2k+1$

    b)   Find two whole numbers, a and b, such that:

        (i) $a^2 - b^2 = 2017$

        (ii) $a^2 - b^2 = 20172017$

    c)   Can all odd numbers be written as a difference of two squares? Justify your answer.

    d)   Carefully expand $(k+2)^2 - k^2$ and find two whole numbers, a and b, such that $a^2 - b^2 = 2016$.

    e)   Can you conclude all even numbers can be written as a difference of two squares?

[10 marks]

2.   Susan has a box of chocolates containing 10 white and 15 dark chocolates. Jane's box of chocolates contains 15 white and 24 dark chocolates.

   a)   What is the ratio of white to dark chocolates in Susan's box?

   b)   What is the ratio of white to dark chocolates in Jane's box?

   c)   Suppose Susan eats one dark chocolate. How many dark chocolates must Jane eat in order that both Susan's and Jane's boxes contain the same ratio of white to dark chocolates?

   d)   Charlotte's box of chocolates also contains both white and dark chocolates. The ratio of white to dark chocolates is 7:9. When Charlotte eats one dark chocolate, the ratio of white to dark chocolates becomes 4:5. How many of each type of chocolate were in Charlotte's box to start with?

[10 marks]

3.   a)   Show that $(x+y)^2 = x^2 + y^2 + 2xy$

Suppose for the rest of the question $x+y=10$ and $xy=2$.

b)   Show that $x^2 + y^2 = 96$

c)   Show that $x^2y + y^2x = 20$

d)   Multiply out $(x+y)(x^2+y^2)$

e)   Hence, or otherwise, find the value of $x^3 + y^3$

[10 marks]

4.   An oil tanker has length 100m, a tug boat has length 17m and a jetski has length 2m.

    a)   If the tanker is stationary and the jetski is travelling alongside it at 35km/h, how long does it take for the front of the jetski to travel the length of the tanker? Give your answer in seconds.

    b)   Suppose instead the tanker is stationary and the tug is travelling alongside it at 12km/h. Initially the rear of the tug is level with the rear of the tanker. How long does it take for the front of the tug to be level with the front of the tanker? Give your answer in seconds.

    c)   Now suppose the tanker is moving at 25km/h and the jetski is travelling alongside it at 35km/h. Initially the rear of both the tanker and jetski are level. How long does it take for the front of the jetski to be level with the front of the tanker? Give your answer in minutes.

[10 marks]

5.   The diagram shows an isosceles right-angled triangle ABC. The sides AC and AB are both 2cm long. The triangles CPQ, QTR and RSB are identical.
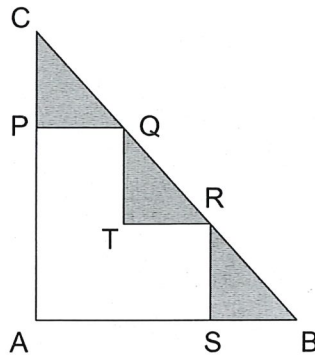
Diagram not to scale.

a)   Calculate the shaded area within triangle ABC.

b)   In the figure below, the centre of the top square is directly above the common edge of the middle two squares, whilst the centres of the middle two squares are directly above the common edges of the lower three squares. Each square has side length 1cm. Find the shaded area.
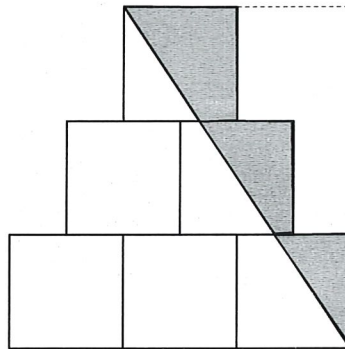
Diagram not to scale.

c)   The figure below is similar (in a non mathematical sense) to that in part
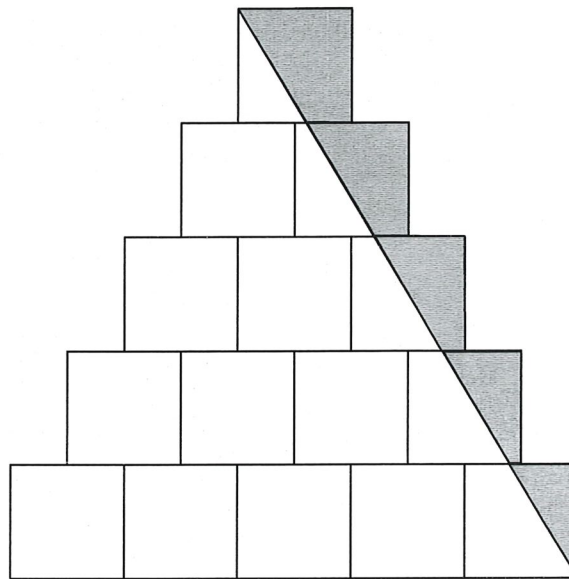   b) except that it has 5 rows. Find the shaded area.

Diagram not to
scale.

[10 marks]

6.   A *gradian* is an alternative measure of angles, rather than the usual measure of degrees. Whilst a right angle contains 90 degrees, it contains 100 *gradians*.

a)   How many *gradians* are equal to 270 degrees?

b)   How many degrees are equal to 70 *gradians*?

c)   Suppose 8 *xadians* equal 200 *gradians*. How many degrees are there in an angle equal to the sum of 3 *xadians* plus 5 *gradians*?

d)   Suppose 5 *yadians* plus 2 *zadians* equals 308 *gradians*, whilst 6 *yadians* subtract 16 *zadians* equals 18 degrees. How many *gradians* equal to the sum of 1 *yadian* plus 1 *zadian*?

[10 marks]

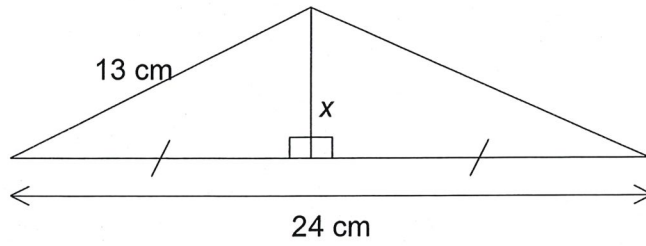7.   a)   The diagram shows an isosceles triangle. Find x.

13 cm

x

24 cm

Diagram not to scale.

b)   The diagram shows a different isosceles triangles. Show that
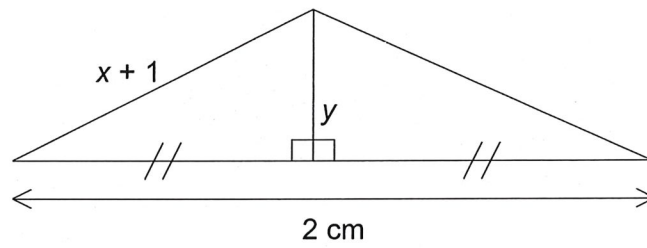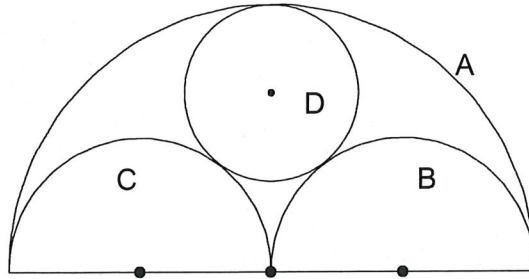
$$y = \sqrt{(x+1)^2 - 1}$$

$x + 1$

y

2 cm

Diagram not to scale.

c) In the following figure, the largest semicircle, A, has radius 2 cm.
Semicircles B and C are the same side and both have radius 1 cm. The
circle D touches semicircles A, B and C and has radius r cm.

(i) Show that $r + \sqrt{(r+1)^2 - 1} = 2$

(ii) Hence show that $r = \dfrac{2}{3}$ cm.

[10 marks]

8. If x is a positive whole number, define $\{x\}$ to be the new whole number that results from moving the units digit of x to the front of x.

    For example, $\{312\}=231$ and $\{589743\} = 358974$.

    a) Hugo writes down a two digit whole number x with tens digit a and units digit b. Show that
    $$\{x\} = \frac{x-b}{10} +10b$$

    b) Next Hugo writes down a three digit whole number y. Find an expression for $\{y\}$ in terms of y and c, the units digit of y.

    c) Hugo finally writes down a six digit number z with a 9 in the unit position. If $\{z\} = 4z$, find the value of z.

[10 marks]

END OF PAPER