

Eton College King's Scholarship Examination 2017

MATHEMATICS A

(One and a half hours)

Candidate Number:.....

Please answer on the paper in the spaces provided.

Please write your candidate number on every sheet.

Answer Question 1 and as many of the other five questions as you can.

Question 1 is worth 50 marks. All other questions are worth 10 marks each.

Show all your working.

The use of calculators is NOT permitted.

ADDITIONAL MATERIALS: *None*

Do not turn over until told to do so.

1. This question is compulsory.

a) This part is about percentages.

i) Find 27% of 32,000. [2]

ii) Richard invests £7,000 in an account that gives 1.2% interest per year. He invests a second amount in a different account that offers 1.5%. Overall after one year he receives 1.25%. How much did he invest in the second account? [3]

b) Solve the following equations, leaving your answers as simplified fractions as appropriate:

i) $37x + 58 = 126 - 65x$ [2]

ii) $\frac{5x-2}{16} - \frac{2x-1}{5} = \frac{1}{80}$ [3]

- c) Two thirds of a theatre audience are children and the rest are adults. A fifth of the children and three eighths of the adults arrive late. What fraction, of those who arrived late, were adults?

[4]

- d) An isosceles triangle ABC has side lengths $AB = AC = 29$ cm and $BC = 42$ cm. Find the area of the triangle ABC.

[5]

- e) Solve the simultaneous equations:

$$7x - 2y = 25$$

$$9x + 5y = 17$$

[4]

- f) Solve the following inequalities, leaving your answers as simplified fractions as appropriate:

i) $5x - 11 > 1 - x$ [2]

ii) $\frac{x}{4} - \frac{x}{3} > \frac{1}{2}$ [3]

- g) Given that $a = \frac{bc}{b-c}$, find the value of a when $b = -13$ and $c = -65$, giving your answer as a decimal. [3]

- h) This part is about lowest common multiples.

i) Find the lowest common multiple of 72 and 90. [3]

- ii) Two numbers 72 and N have a lowest common multiple 720. What is the smallest possible value of N? [3]

i) By what do I need to

- i) multiply $10x^2$ to get $\frac{15x}{y}$; [2]

- ii) divide $15x$ to get $\frac{10y}{x}$? [2]

j) Given that $473 \times 271 = 128183$, calculate the following:

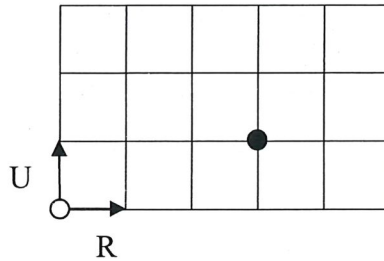
- i) 473×171 [2]

- ii) 473×271271 [3]

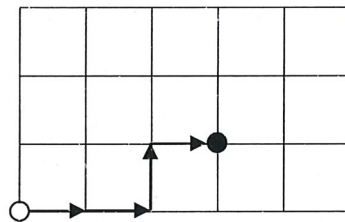
- k) A regular polygon with n sides has an interior angle of x° . Show that a regular polygon with $2n$ sides has an interior angle of $(180 + \frac{1}{2}x)^\circ$. [4]

[Total for Question: 50]

2. The diagram shows a 5 by 3 grid. A route is a set of moves, starting at the white dot and ending at the black dot, where each move takes you to an adjacent corner on the grid. Move R takes you to the grid point to the right of where you start, move U takes you to the grid point above where you start, as illustrated on the diagram.

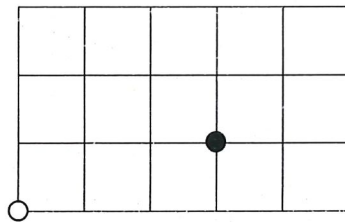


The route R, followed by R, followed by U, followed by R, is illustrated below.



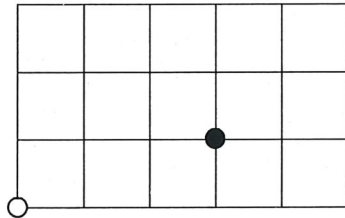
- a) Illustrate on the diagram below the route U, followed by R, followed by R, followed by R.

[2]

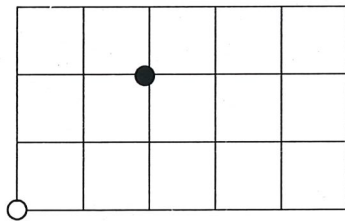


- b) Explain why there are 4 different routes from the white dot to the black dot on the diagram below.

[2]

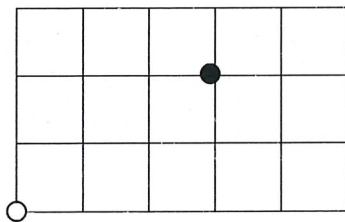


- c) Find the number of routes on this diagram.



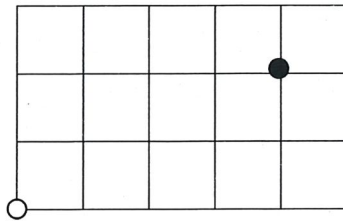
[2]

- d) Use your answers to b) and c) to find the number of routes on this diagram.



[2]

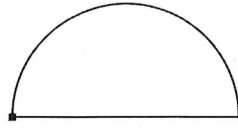
e) Find the number of routes on this diagram.



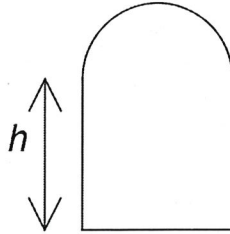
[2]

[Total for Question:10]

3. In the diagram below, shape A is a semicircle and shape B comprises a semicircle and rectangle. The radius of the semicircle in shape A is 1 cm more than the radius of the semicircle in shape B. The height of the rectangle in shape B is h cm.



Shape A



Shape B

Diagram not to scale.

- a) Suppose the two shapes have the same perimeter. Show that $h = \frac{\pi + 2}{2}$. [5]

- b) Suppose INSTEAD that the area of the two shapes are equal and $h = 3\pi$. Find the radius of the semicircle in shape A. [5]

[Total for Question:10]

4. This question is about angles and lengths.

- a) In the diagram below, angle ABD is a right angle. The lengths AC and BC are equal and the lengths BD and DE are equal. Giving reasons, find the angle ADE. [4]

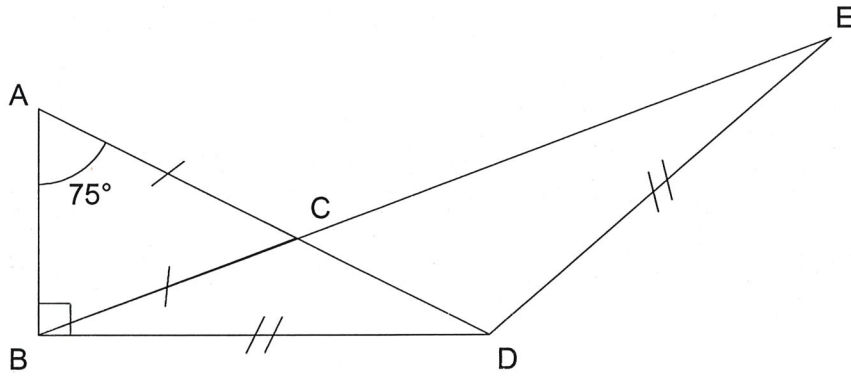


Diagram not to scale.

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- b) In the diagram below, $PQ = QR$ and $PR + PQ = QS$. Furthermore PR and QS are parallel. Show that angle QSR is half angle QPR. [6]

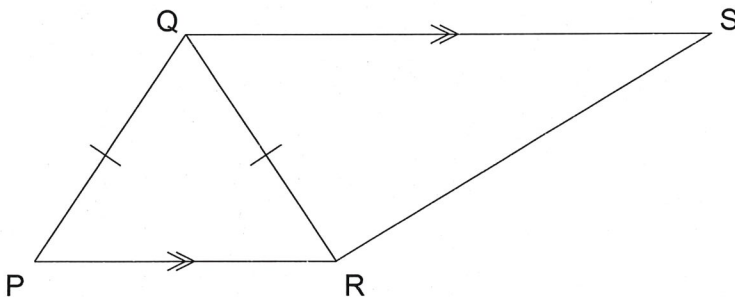


Diagram not to scale.

[Total for Question:10]

5. This question is about reversing digits in numbers e.g. starting with a number 1347 and replacing it with 7431.
- a) Adam starts with a two digit number and reverses its digits. He then adds the two numbers together to get an answer.
- i) Show that if he starts with the number with tens digit a and units digit b , his answer will be $11(a + b)$. [2]
- ii) For how many different starting numbers could his answer be 143? [2]
- b) Ben starts with a 4 digit number and reverses its digits. He then adds the two numbers together.
- i) Show that his answer is always a multiple of 11. [3]
- ii) Show that his answer will be a multiple of 143 exactly when the two middle digits of the original number add to either 0 or 13. [3]

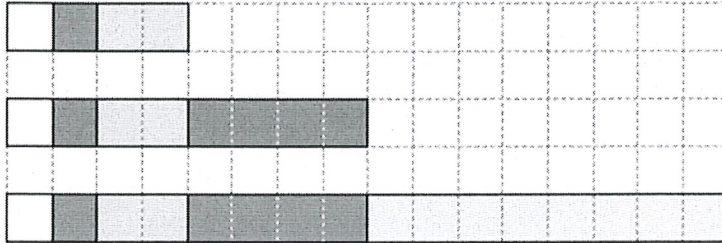
[Total for Question:10]

6. The diagrams below show that

$$1 + 2 = 4 - 1$$

$$1 + 2 + 4 = 8 - 1$$

$$1 + 2 + 4 + 8 = 16 - 1$$



a) Draw a diagram (not to scale) to show that $1 + 2 + 4 + 8 + 16 = 32 - 1$

[2]

b) Explain why $1 + 2 + 4 + 8 + \dots + 2^{2016} = 2^{2017} - 1$

[2]

A set of numbers is said to be sum-free if it is not possible to write any number in the set as the sum of other numbers in the set, using the other numbers at most once. For example, the set $\{1, 2, 4, 8, 11\}$ is not sum-free as $11 = 1 + 2 + 8$, but $\{1, 2, 4\}$ is sum-free.

c) Show that the set $\{1, 2, 4, 8, 16\}$ is sum-free.

[2]

d) Is the set $\{1, 2, 4, 8, 16, \dots, 2^{2017}\}$ sum-free?

[2]

e) Mark starts writing down the numbers 1, 2017, 4035, 6052 etc. If he continues this, will he always have a sum-free set?

[2]

[Total for Question: 10]

END OF PAPER