

# Eton College King's Scholarship Examination 2015

## MATHEMATICS A

(One and a half hours)

*Answer Question 1 and as many of the other five questions as you can.*

*Question 1 is worth 50 marks. All other questions are worth 10 marks each.*

*Show all your working.*

*The use of calculators is NOT permitted.*

*Remember to write your candidate number on every sheet of answer paper used.*

**Do not turn over until told to do so.**

1. This question is compulsory.

(a) Given that  $a = -3$  and  $b = 5$ , calculate the following:

(i)  $b^2 - a^2$

[2]

(ii)  $\frac{2b^2}{a-2}$  [3]

(b) Calculate the following, leaving your answers as simplified mixed fractions:

(i)  $10\frac{5}{8} - 3\frac{1}{40}$  [3]

(ii)  $8 \div 1\frac{5}{11}$  [3]

(c) Simon wishes to sell his car for £3,200.

(i) If Simon agrees to give me a 15% discount, how much would I have to pay for the car? [2]

(ii) If I buy the car at this discount and then sell it back to Simon for 15% more than I paid for it, how much do I sell it for? [2]

(d) Evie is not very good at using a protractor. When she measures an angle, she always ends up with a value which is  $180^\circ$  minus the correct value (e.g. if the angle is  $40^\circ$ , she measures it as  $140^\circ$ ). If she measures the angles of a triangle and adds them up, what answer will she get? [3]

(e) Solve the following equations, giving your answers as simplified fractions:

(i)  $x - 2(7 - 3x) = 4$  [3]

(ii)  $\frac{2}{3}(y + 2) - \frac{y}{6} = \frac{41}{24}$  [4]

(f) Solve the simultaneous equations:

$$5x - 2y = 12$$

$$7x + 3y = 11$$

[4]

(g) (i) The mean average of 3 numbers is 4.7. If two of the numbers are both 3.1, what is the third number? [2]

(ii) The smallest number in a group of 3 numbers is 2.1 less than the middle number and 4.5 less than the biggest number. How much larger than the smallest number is the mean average of all 3? [3]

(h) Solve the following inequalities:

(i)  $4 + x < \frac{1}{2}x$  [3]

(ii)  $y - (1 - y) < 1 + y$  [3]

(i) Iris has £5,000 to invest. She invests some of it in a company called Safehouse and the rest in a company called Riskybusiness. After one year her investment in Safehouse has increased by 20% but her investment in Riskybusiness has decreased by 20%. If her total investment after one year is now worth £5,300, use algebra to find how much she initially invested in Safehouse. [4]

(j) (i) What do I add to  $3 - 2x$  to get  $x - 2$ ? [2]

(ii) By what do I multiply  $\frac{2}{x}$  to get  $\frac{x}{2}$ ? [2]

(iii) By what do I divide  $\frac{y}{x}$  to get  $\frac{x}{y}$ ? [2]

2. (a) A triangle PQR has two sides PQ and QR which are equal in length. Given that the angles at P, Q and R are  $(x + y)^\circ$ ,  $(x + 2y)^\circ$  and  $(3x - y)^\circ$  respectively, find  $x$  and  $y$ . [4]

(b) The diagram shows an isosceles triangle ABC in which  $AB = AC$  and angle  $BAC = 20^\circ$ . The points X and Y are on the sides AB and AC respectively, and  $BC = CX = CY$ .  
 (i) Giving reasons, calculate the angles ABC and XCB. [3]  
 (ii) Prove that  $XY = BC$ . [3]

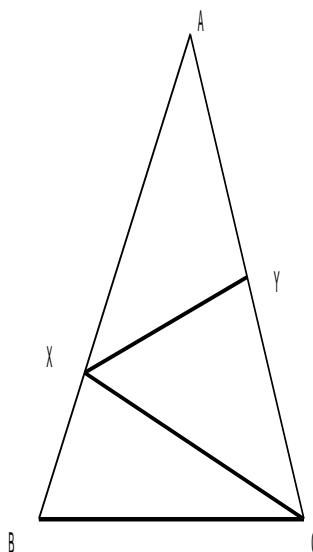


Diagram Not to scale

3. (a) Show that  $(10,000 + a)^2 = 100,000,000 + 20,000a + a^2$ . [2]
- (b) Show there are 4 digits which are 0 in the square of ten thousand and seven. [2]
- (c) A quattuordecillion is  $10^{45}$ . How many zeroes are there in the square of one quattuordecillion and seven. [3]
- (d) If the number  $x$  is two fewer than a quattuordecillion, how many zeroes are there in  $x^2$ ? [3]

4. (a) In the diagram below, A, B and C lie on a straight line, and angle DAB is  $90^\circ$ .  
 $DC = 52$  cm,  $DB = 25$  cm and  $AC = 48$  cm.  
 (i) Find the length DA. [3]  
 (ii) Find the length AB. [2]

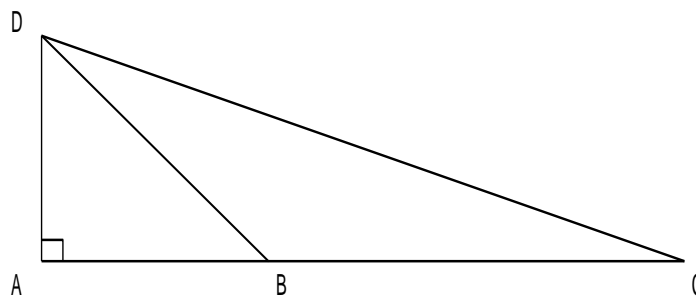
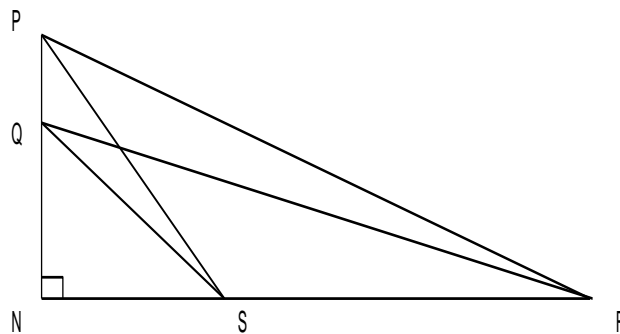


Diagram Not to scale

- (b) In the diagram below, the points P, Q and N lie on a straight line and the points R, S and N lie on a second straight line. The angle PNR is  $90^\circ$ .  
 Show that  $PR^2 + QS^2 = PS^2 + QR^2$ . [5]



5. A derangement is a re-ordering of the members of a set such that none of the members appear in their original position. For example BCAED is a derangement of ABCDE, but BCEDA is not (as the D remains in its original position).

- (a) Write down both derangements of ABC. [1]
- (b) (i) Write down all the derangements of ABCD which start with a B [2]  
 (ii) Show there are 9 derangements of ABCD. [2]
- (c) There are 11 derangements of ABCDE which start with a B. How many derangements are there of ABCDE in total? [2]

An almost-derangement is a re-ordering of the members of a set such that exactly one of the members appears in its original position. For example BCEDA is an almost-derangement of ABCDE.

- (d) How many almost-derangements of ABCDEF are there? [3]

6. (a) For any two whole numbers  $n$  and  $m$ , we define  $n \nabla m$  as follows:

$$n \nabla 1 = 1;$$

$$n \nabla m = n \nabla (m - 1) + n.$$

For example,

$$5 \nabla 1 = 1,$$

$$5 \nabla 2 = 5 \nabla 1 + 5 = 6;$$

$$5 \nabla 3 = 5 \nabla 2 + 5 = 11.$$

- (i) Show that  $7 \nabla 4 = 22$ . [2]
- (ii) Find an expression for  $n \nabla 6$  in terms of  $n$ . [2]

(b) For any two whole numbers  $n$  and  $m$ , we define  $n \angle m$  as follows:

$$n \angle 1 = n;$$

$$n \angle m = n \angle (m - 1) + m.$$

For example,

$$5 \angle 1 = 5,$$

$$5 \angle 2 = 5 \angle 1 + 2 = 7$$

$$5 \angle 3 = 5 \angle 2 + 3 = 10.$$

- (i) Show that  $5 \angle 7 = 32$ . [2]
- (ii) For how many different values of  $n$  does the sequence  
 $n \angle 1, n \angle 2, n \angle 3, n \angle 4, \dots$   
 contain exactly two numbers between 95 and 100 inclusively. [4]

